Lecture 22 – Weiss model of ferromagnetism

Recall example of paramagnetism with independent, non-interacting spins (assume spin 1/2)

In a magnetic field B, each spin has energy:

$$E = -mB \quad \text{with} \quad m = \mu_B s \quad , \quad s = \pm 1 \; (\uparrow \text{ or } \downarrow)$$

$-mB$: favors aligning with B

m: magnetic dipole moment

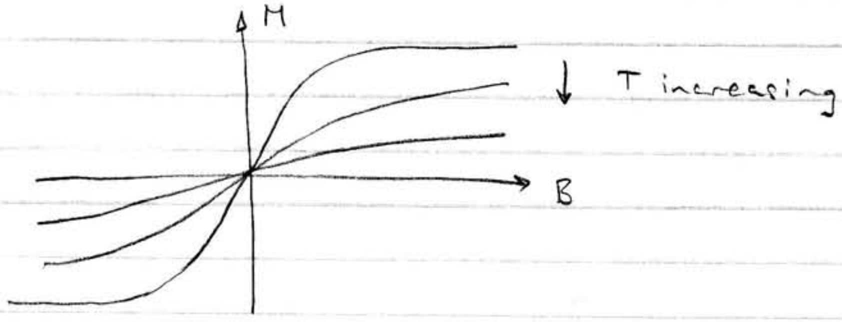
μ_B: Bohr magneton

$$Z = e^{\beta \mu_B B} + e^{-\beta \mu_B B}$$

$$\langle m \rangle = \mu_B \langle s \rangle = \mu_B \frac{1 \cdot e^{\beta \mu_B B} - 1 \cdot e^{-\beta \mu_B B}}{Z} = \mu_B \tanh \frac{\mu_B B}{k_B T}$$

For a material with density n of spins, the magnetization density M is:

$$M(B,T) = n \langle m \rangle = n \mu_B \langle s \rangle = n \mu_B \tanh \frac{\mu_B B}{k_B T}$$



For small B $\quad M(B,T) \approx n \mu_B^2 \frac{B}{k_B T}$

Define susceptibility $\quad \chi \equiv \left. \frac{\partial M}{\partial H} \right|_{H \to 0} = \frac{n \mu_B^2}{k_B T} \propto \frac{1}{T}$ $\quad$ Curie Law

Notice that for paramagnets $M \to 0$ as $B \to 0$.
In ferromagnets (ex: iron), there is a spontaneous magnetization
$M \neq 0$ even when $B = 0$

Our theory so far cannot explain this. We need interactions between spins to get ferromagnetism

"Exchange" interaction (due to quantum mechanics) leads to tendency for neighboring spins to align:

$$E_{exch} = -2J_{ij}\, s_i s_j \qquad \text{for a pair of spins } i,j$$

$$\therefore \quad E_{tot} = -\sum_i \mu_B B s_i - \underbrace{\frac{1}{2}\sum_{i,j} 2J_{ij}\, s_i s_j}_{\substack{\text{exchange interaction is}\\ \text{short range - only nearest}\\ \text{neighbor spins interact}}}$$

$$-J \sum_{i,j}{}' s_i s_j \qquad \left(\Sigma' = \begin{array}{c}\text{sum over}\\ \text{nearest}\\ \text{neighbors}\end{array} \right)$$

This is called the Ising model, only solvable in special circumstances (1-D, 2-D with $B = 0$)

We will solve it approximately using Mean Field Theory
Like Van der Waals gas, replace interaction term with average

For ith spin $\quad E_i \cong -\mu_B B s_i - \underbrace{2J \sum_j{}' \langle s_j \rangle}_{2J\langle s\rangle N_{n.n.}} s_i$

$2J\langle s\rangle N_{n.n.} \propto M$

$N_{n.n.}$: # nearest neighbors

M: since $M = n\mu_B\langle s\rangle$

$\therefore \quad E_i \cong -\mu_B B s_i \quad -\mu_B \lambda M s_i \quad = \quad -\mu_B (B + B_{eff}) s_i$

Average interaction term acts like an <u>effective B-field</u> $B_{eff} \equiv \lambda M$ due to neighboring spins that tends to align with spin

Solution is easy → take paramagnet and add B_{eff} !

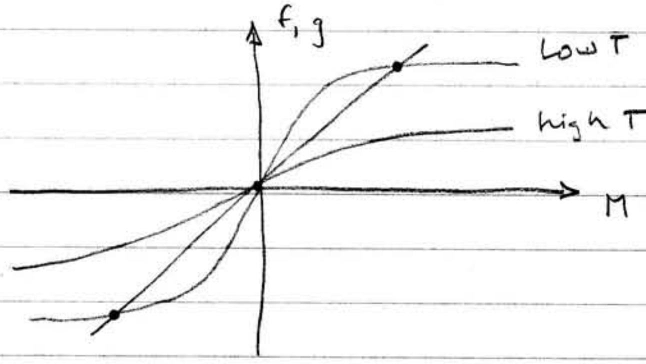
$$M = n\mu_B \tanh\left(\frac{\mu_B(B + B_{eff})}{k_B T}\right) \qquad B_{eff} = \lambda M$$

Consider case where external <u>B field = 0</u> :

$$M = n\mu_B \tanh\left(\frac{\mu_B \lambda M}{k_B T}\right)$$

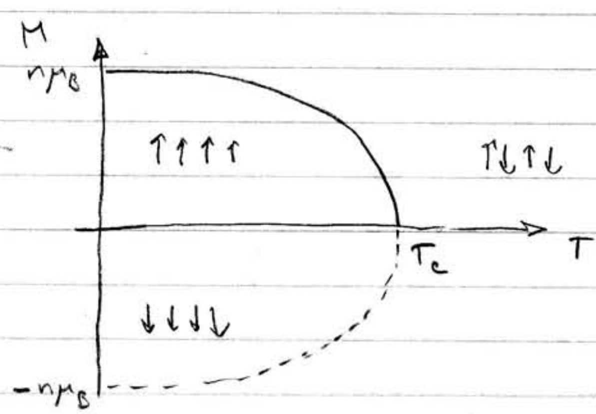
← transcedental equation M is on both sides. Solve graphically

Define $f(M) = M \qquad g(M) = n\mu_B \tanh\left(\frac{\mu_B \lambda M}{k_B T}\right)$



At high T, only one sol'n $M = 0$

At low T, 3 sol'ns $\underbrace{M > 0,\ M < 0}_{\text{stable}}$, $\underset{\text{unstable}}{M = 0}$

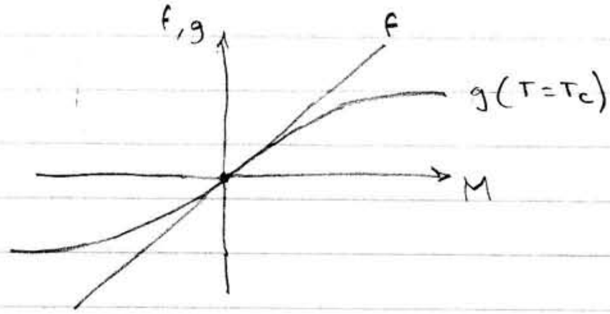


Below some temperature T_c get $M \neq 0$ despite $B = 0$ spontaneous magnetization

$M > 0$ spins up $\quad M < 0$ spins down

In the absence of an external field B, either state ($\uparrow\uparrow\uparrow$ or $\downarrow\downarrow\downarrow$) is equally likely, we say symmetry is spontaneously broken

How do we find T_c?



This will occur when slopes of $f(M)$ $g(M)$ at $M=0$ are equal

$$f'(M=0) = 1$$
$$g'(M=0) = n\mu_B\left(\frac{\mu_B\lambda}{k_B T_c}\right) = 1$$

$$\therefore \boxed{T_c = \frac{n\mu_B^2\lambda}{k_B}}$$

Curie temperature
T at which system transitions from paramagnet to ferromagnet

Ex: N_i: $T_c = 631K$ gives $\lambda \sim 2000$

$M(T=0) \approx 0.17$ Tesla $\therefore$ $B_{eff} = \lambda M \sim 10^3$ Tesla

This is a huge field, much larger than B field acting on 1 spin due to neighboring spins.

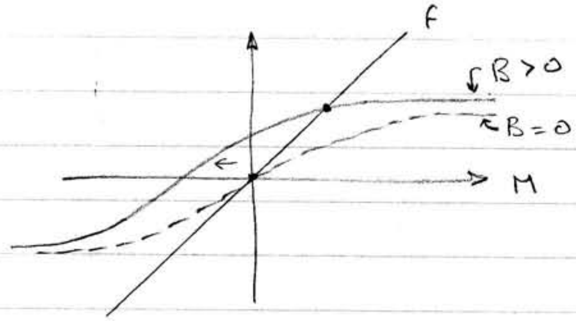
Estimate $B_{actual} \sim \frac{\mu_B}{a^3} \sim 0.1\,T \ll 10^3\,T$ a = lattice spacing

B_{eff} is not classical dipole-dipole coupling
Interaction is quantum mechanical effect, much stronger than that expected classically

What happens to M in external field B?

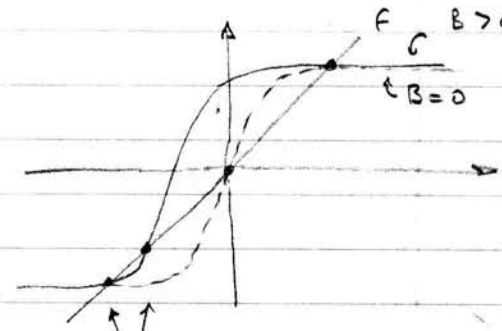
$$M = n\mu_B \tanh\left(\frac{\mu_B B}{k_B T} + \frac{\mu_B \lambda M}{k_B T}\right)$$

For $T \geq T_c$



Get magnetization depending on B (paramagnet)

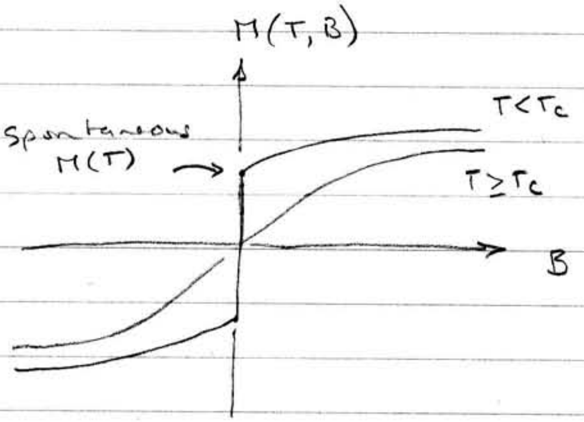
For $T < T_c$



unstable sol'ns

magnetization spontaneous weakly dependent on B (ferromagnet)

Plot M vs. B for different T



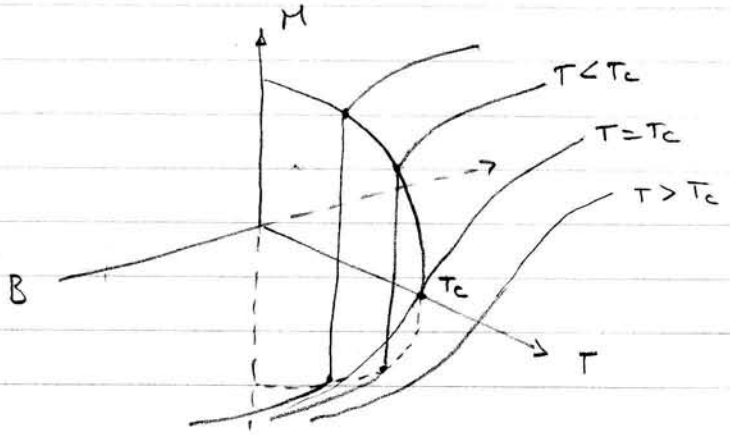
Note that susceptibility is

$$\chi = \left.\frac{\partial M}{\partial H}\right|_{H=0} = \text{slope of } M \text{ vs. } B \text{ at origin}$$

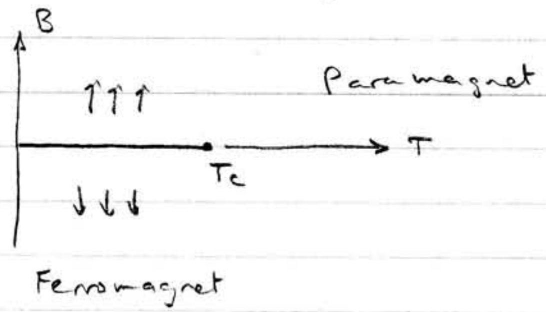
for $T \geq T_c$ $\quad \chi \geq 0$

$T < T_c$ $\quad \chi \to \infty$

In 3-D $M(T, B)$ is



Look down along M axis ⇒ get phase diagram



T_c is a critical point

This transition is second-order ⇒ no latent heat
entropy changes continuously across transition, unlike
gas → liquid transition (away from critical pt.)

Behavior near critical pt. ($T \lesssim T_c$, $B \approx 0$)
Systems near critical pt. exhibit universal behavior.
Look at $M(T)$, $\chi(T)$ near T_c

$M(T)$ is small near T_c, expand tanh to lowest order

$$M = n\mu_B \tanh\left(\frac{\mu_B \lambda M}{k_B T}\right) \qquad T_c = n\mu_B^2 \frac{\lambda}{k_B}$$

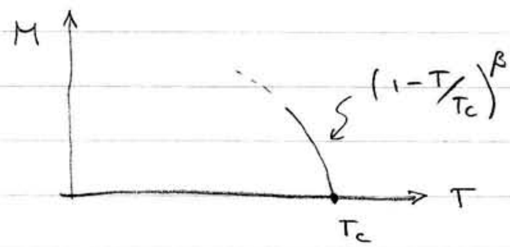
Define $m = \dfrac{M}{n\mu_B}$ so $m = \tanh\left(\dfrac{m T_c}{T}\right)$

$\tanh x \approx x - \dfrac{x^3}{3} + \cdots$ for $x \ll 1$

$\therefore \; m \approx m\dfrac{T_c}{T} - \dfrac{1}{3}\left(m\dfrac{T_c}{T}\right)^3 + \cdots$ valid for $T \lessapprox T_c$

$$1 \approx \frac{T_c}{T}\left(1 - \frac{1}{3}\left(m\frac{T_c}{T}\right)^2\right)$$

$$\therefore \quad m = \frac{T}{T_c}\sqrt{3}\left(1 - T/T_c\right)^{1/2}$$



$M \propto \left(1 - T/T_c\right)^\beta \qquad \beta = \frac{1}{2}$

β = "critical exponent"

Now look at susceptibility χ
For a small external field B

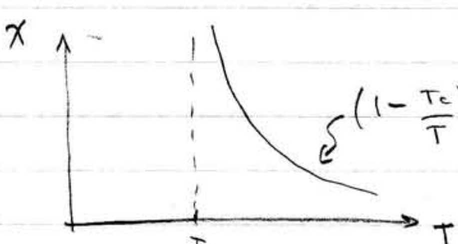
$$M = n\mu_B \tanh\left(\frac{\mu_B B}{k_B T} + \frac{\mu_B \lambda M}{k_B T}\right)$$

$$\Rightarrow \; m = \tanh\left(\frac{\mu_B B}{k_B T} + m\frac{T_c}{T}\right)$$

$$\approx m\frac{T_c}{T} + \frac{\mu_B B}{k_B T} + \cdots \qquad \Rightarrow \quad m\left(1 - \frac{T_c}{T}\right) = \frac{\mu_B B}{k_B T}$$

$$M = n\mu_B m = \frac{n\mu_B^2 B}{k_B}\,\frac{1}{T - T_c}$$

"Curie-Weiss law"



$\therefore \; \chi \propto \left(1 - \dfrac{T_c}{T}\right)^{-\gamma} \qquad \gamma = 1$

Diverges at $T \to T_c$